



ACCELERATOR EXPERIMENT: Third Order, Vertical Resonance  
in the Main Ring

Experimentalists: R. Stiening, T. Wilson (Exp. #224)

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Third order, vertical resonance has been found in the Main Ring at the injection energy of 8 GeV. The resonance is of the type

$$3\nu_y = 61$$

To kill this resonance, twelve air-core, skew sextupoles have been installed at all stations #14 and #39. The field integrated across the length of each sextupole is

$$\int B dl = 4.28 y^2 \frac{\text{Gauss} \times \text{cm}}{\text{Ampère}}$$

The following setting just eliminates the resonance

Section Station	A	B	C	D	E	F
#14	48	-16	-64	-48	16	64
#39	-66	-41	22	66	41	-22

These numbers are the p.s. units, where 127 units = 5 Ampères. The driving factor of the resonance is

$$F = \int_0^C \beta_y^{\frac{3}{2}}(s) B''(s) e^{i61\phi(s)} ds \quad (1)$$

A. We calculate F from the setting of the air-core sextupoles.

All the sextupoles are located at positions where  $\beta_y = 96$  m. The phase difference between stations #39 and #14 of the same section is 0.67127. The phase difference between two homologous, next to each other's stations is, obviously,  $\pi/3$ .

We obtain, then,

$$|F| = 11 \times 10^3 \text{ kgauss} \times \text{m}^{1/2}$$

B. A possible explanation of this vertical resonance is the distortion of the plane of symmetry in the quadrupoles (TM-387). The curvature of the horizontal symmetry plane is measured by the kink angle. The rms of the value of the angle, calculated on the basis of the magnetic data information of 180 quadrupoles, is of about 20 mrad. We calculate then the expectation value of the driving factor (1). Most of the contribution is from F-quads. We have

$$\langle F \rangle = \beta_{\max}^{\frac{3}{2}} \ell \sqrt{\frac{M}{2}} \langle B'' \rangle$$

where  $\ell$  is the length of a quadrupole and M the total number of the quadrupoles.

It is

$$\beta_{\max}^{\frac{3}{2}} = 10^6 \text{ cm}^{\frac{3}{2}}$$

$$\langle B'' \rangle = 0.20 \text{ G/cm}^2 \quad (\text{TM-387})$$

$$\sqrt{M/2} = 10$$

$$\ell = 200 \text{ cm}$$

from which

$$\langle F \rangle = 40 \times 10^3 \text{ Kgauss} \times \text{m}^{1/2} .$$

This number is larger (but not too much larger) than the measured one. We believe, then, we have found an explanation for the third order vertical resonance.

C. The width of the resonance. This is calculated from the condition of stability that the beam emittance,  $E$ , must be less than the area of the stable region in the phase space. We have

$$\Delta v = 5.2643 \frac{|F|}{48\pi B_p} \sqrt{E} \quad (\text{total width}) .$$

At 8 GeV in the Main Ring, it is

$$B_p = 296.5 \text{ kgauss} \times \text{m}$$

$$E = 10\pi \cdot 10^{-6} \text{ m} \times \text{rad}$$

from which

$$\Delta v = 0.007 .$$

A.G. Ruggiero